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Maxwell Equations, Nonzero Photon Mass, and Conformal Metric Fluctuation

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Maxwell equations are studied for a vacuum with nonzero conductivity coefficient. The loss of energy of a photon during its propagation through this vacuum is calculated and the nonzero rest mass of the photon is shown to be related to the conductivity coefficient of the vacuum. The dissipative mechanism is investigated considering a conformally fluctuating metric in the Einstein equation. Possible astrophysical consequences are discussed.

1. INTRODUCTION

Maxwell's equations have been studied in a vacuum with a nonzero conductivity coefficient, i.e., with $\sigma \neq 0$. The nonzero conductivity coefficient gives rise to a dissipative term in the field equation. In this case if we consider the propagation of a photon through this type of vacuum, the photon acquires a mass at cosmological scale. In fact, due to the presence of the dissipative term in the field equation, the photon loses energy during the propagation through this vacuum. This dissipation can be related to the conformal fluctuation of the metric of the background space-time. Taking a conformally fluctuating metric in the Einstein equation for the gravitational field (Sinha and Roy, 1987), we have constructed the average metric tensor of the background space-time in the vacuum. After averaging over the ensemble of the random scalar field, the fluctuation of the metric generates a term like the cosmological constant term Λ in the Einstein equation. There is a parameter ξ $(0 < \xi \leq 1)$ in our framework which plays a significant role in determining Λ as well as in describing the behavior of fluctuations. The parameter ξ is related to the correlation of the space-time derivatives of the scalar field.

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On the other hand, if we calculate the propagation of the photon field in a vacuum with refractive index $n \neq 1$ and $\sigma \neq 0$, then we can construct an effective metric tensor of the background space-time containing the refractive index. Now we can compare this effective metric tensor with the average metric tensor from the Einstein equation. Then we get a relationship between the refractive index and the parameter ξ.

The refractive index n becomes real or imaginary depending on the value of ξ. For imaginary refractive index, i.e., for certain values of ξ, the photon acquires mass during its propagation through the vacuum. For particular values of ξ and of real refractive index, the cosmological term Λ vanishes and we get a positive-definite metric tensor similar to that considered by Dohrn *et al.* (1985) in describing a conservative diffusion process. The existence of a conservative diffusion process plays a significant role in the stochastic quantum mechanics proposed by Nelson (1985). It is generally believed that quantum fluctuations should be considered as nondissipative in nature. This is unlike other statistical fluctuations in physics in the sense that they are considered as dissipative in nature (Smolin, 1986).

In Section 2 we study the relation between the nonzero rest mass of the photon and the conductivity coefficient of the vacuum. In Section 3 we study the Einstein equation for the gravitational field with a conformally fluctuating metric and the nature of the situation on a microscopic scale as well as on a cosmological scale. In Section 4 we describe possible astrophysical consequences, such as the relation between the massive photon and the Hubble constant (Fuli, 1975). Finally, an estimate of the conductivity coefficient σ of the vacuum is made which can be tested in laboratory experiments.

2. MAXWELL EQUATIONS AND NONZERO PHOTON MASS IN VACUUM

If we endow the vacuum with nonzero conductivity, i.e., $\sigma \neq 0$ in the vacuum, Maxwell's wave equations should be rewritten in the form

$$\operatorname{div} \mathbf{E} = 0 \qquad \operatorname{curl} \mathbf{H} = \sigma \mathbf{E} + \varepsilon_0 \kappa_e \frac{\partial \mathbf{E}}{\partial t} \tag{1a}$$

$$\operatorname{div} \mathbf{H} = 0 \qquad \operatorname{curl} \mathbf{E} = -\mu_0 \kappa_m \frac{\partial \mathbf{H}}{\partial t} \tag{1b}$$

where ε_0 denotes the vacuum's dielectric constant, μ_0 denotes the vacuum's permeability constant, κ_e is the relative dielectric constant, and κ_m is the relative permeability constant.

Now,

$$\nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E}$$

which together with Maxwell's equations gives

$$\nabla^2 \mathbf{E} = -\frac{\varepsilon_0 \kappa_e \kappa_m}{c^2} \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} + \sigma \mu_0 \kappa_m \frac{\partial \mathbf{E}}{\partial t} \tag{2}$$

This equation is not time-reversal-invariant. The second term on the right-hand side indicates that there will be a dissipation of energy during the propagation of a photon.

If we consider only plane waves in the z direction,

$$E_x = b\, e^{i\omega(t - z/v)}, \qquad H_y = b\left(\frac{e_0 \kappa_e}{\mu_0 \kappa_m}\right)^{1/2} e^{i\omega(t - z/v)}$$

putting $q = 1/v$ and the plane wave solution of $\mathbf{E}$ in (2), we get

$$q^2 = \frac{\kappa_e \kappa_m}{c^2}\left(1 - \frac{i\sigma}{\varepsilon_0 \kappa_e v}\right) \varepsilon_0 \mu_0 \tag{3}$$

Thus, it is evident from the above equation that q can be taken as complex in nature, having the form of $\alpha - i\beta$, where α and β are real and are given by

$$\begin{aligned} \alpha &= 1 + \frac{1}{8}\left(\frac{\sigma}{\varepsilon_0 \kappa_e}\right)^2 \frac{1}{\omega^2} + O\left(\frac{\sigma^4}{\omega^4}\right) \\ \beta^2 &= \frac{1}{2}\left(\frac{\sigma}{\varepsilon_0 \kappa_e}\right)^2 \frac{1}{\omega^2} \end{aligned} \tag{4}$$

for $\sigma/\omega \to 0$.

Then the velocity defined by v in equation (3) will give rise to a complex refractive index n in the vacuum. The velocity defined by $v = 1/\alpha$ is the phase velocity of propagation of the disturbance through the underlying vacuum. Henceforth it will be denoted as v_p. So

$$v_p = n\left(1 - \frac{1}{8} - \frac{\sigma^2}{(\varepsilon_0 \kappa_e)^2 n^4 \omega^2}\right)$$

and the group velocity can be written as

$$v_g = \frac{\partial \omega}{\partial k} \qquad \text{where} \quad v_p = \frac{\omega}{k}$$

Hence

$$v_g = n^2 \frac{k}{\omega} = n\left[1 + \frac{1}{4}\frac{\sigma^2}{(\varepsilon_0 \kappa_e)^2 n^4 \omega^2}\right]^{1/2}$$

where the dispersion law gives $k^2 = \omega^2 + \frac{1}{4}\sigma^2$.

Taking the values of α and β and $n=1$, we can show E_x and E_y to be proportional to

$$\exp(-\omega\beta z)\exp(t - \alpha z)$$

Then we arrive at the following results:

(a) Plane waves are progressively damped with the factor $\exp(-kz)$, where $k = \omega\beta$.
(b) The phase velocity v_p of propagation of the wave is $1/\alpha$ and varies with frequency.

The fact that equation (2) is not time-reversal-invariant is due to the second term on the right-hand side of (2), which indicates that there will be a dissipation of energy during the propagation of a photon.

Taking v_g as the velocity of a photon in de Broglie's relation (de Broglie and Vigier, 1972),

$$E = h\nu = (1 - v_g^2/c^2)^{1/2} \tag{5}$$

and $\omega = 2\pi\nu$, we get the mass of the photon as

$$m_\gamma^2 = \hbar^2\omega^2(1 - n^2) - \frac{\sigma^2\hbar^2}{4n^2(\varepsilon_0\kappa_e)^2} \tag{6}$$

for the Maxwell vacuum with an imaginary refractive index $n^2 = -m^2$ (say) and

$$m_\gamma^2 = \hbar^2\omega^2(1 + m^2) + \frac{\sigma^2\hbar^2}{4(m^2\varepsilon_0^2\kappa_e^2)} \tag{7}$$

It appears from the relation (6) that the mass of the photon will depend on the frequency of the electromagnetic wave. The mass will be zero when

$$\hbar^2\omega^2(1 - n^2) = \frac{\sigma^2\hbar^2}{4n^2(\varepsilon_0\kappa_e)^2}$$

or

$$\omega^2 = \frac{\sigma^2}{4n^2(1 - n^2)(\varepsilon_0\kappa_e)^2}$$

Thus, the photon with every possible frequency will not gain mass during its propagation through this type of vacuum for fixed n and fixed σ. Hence, for fixed n and σ the mass of the photon is dependent on the frequency during its propagation through this type of vacuum. Moreover, for a complex refractive index the mass will be large. Several authors (Fuli, 1975; Vigier, 1990) also calculated the effective photon mass as

$$m_\gamma = \frac{hH}{2c^2} \simeq 10^{-65}\ \text{g} \tag{8}$$

where H is the Hubble constant.

They tried to relate it to an explanation of the redshift mechanism on the cosmological scale. But it requires a detailed analysis to find such a relationship and use it for an alternate mechanism for the redshift. For small ω and a complex refractive index, say $n^2 = -1$, we have also $m_\gamma \sim (1/2)\sigma/\hbar\varepsilon_0\kappa_e$ from equation (6). Now we can find the structure of such a vacuum where the photon loses energy during its propagation and becomes massive.

Equation (2) can be written as

$$\nabla^2\mathbf{E} = n^2\frac{\partial^2\mathbf{E}}{\partial t^2} + \mu_0\kappa_m\sigma\frac{\partial\mathbf{E}}{\partial t} \tag{9}$$

with $\kappa_e\kappa_m = 1$, $n^2 = \varepsilon_0\mu_0$, and $c = 1$.

Putting $\mathbf{E} = -\partial\mathbf{A}/\partial t - \nabla\phi$ in (9), we get

$$\begin{aligned}\nabla^2\mathbf{A} - n^2\frac{\partial^2\mathbf{A}}{\partial t^2} &= \mu_0\kappa_m\sigma\frac{\partial\mathbf{A}}{\partial t} + \mu_0\kappa_m\sigma\,\nabla\phi \\ &= \mu_0\kappa_m\sigma\left(\frac{\partial\mathbf{A}}{\partial t} + \nabla\phi\right) \\ &= -\mu_0\kappa_m\mathbf{J}\end{aligned} \tag{10}$$

where

$$\mathbf{J} = \sigma\mathbf{E} = \sigma\left(-\frac{\partial\mathbf{A}}{\partial t} - \nabla\phi\right)$$

and

$$\nabla^2\phi - n^2\frac{\partial^2\phi}{\partial t^2} = 0 \tag{11}$$

Using the plane wave solution, we get the following dispersion equation, written in a covariant form:

$$(|k|^2 - n^2 k_0^2)\mathbf{A}^\mu(k) = \kappa_m \mu_0 \left(g^{\mu\nu} + \frac{n^2 - 1}{n^2} \eta^\mu \eta^\nu \right) \mathbf{J}_\nu(k) \tag{12}$$

where

$$\mathbf{J} = (0, \sigma \mathbf{E}), \qquad \mathbf{K} = (k^0, \mathbf{k})$$

Note that we get exactly the same dispersion as derived by Schwinger *et al.* (1976) for a medium with a refractive index $n = \hbar 1$. Here η^μ is the unit timelike vector and $\eta = (0, 1)$ for the medium at rest. Hence the effective metric tensor of the underlying vacuum having $\sigma = 0$ and $n = 1$ can be expressed as

$$G^{\mu\nu} = g^{\mu\nu} + \frac{n^2 - 1}{n^2} \eta^\mu \eta^\nu \tag{13}$$

We have already shown that if a photon propagates through the vacuum it loses energy and this dissipation can be related to the conformal metric fluctuation of space-time.

3. CONFORMAL METRIC FLUCTUATION AND EINSTEIN EQUATION

In this case the vacuum has been characterized by the fluctuation of the space-time metric. This fluctuation diminishes as the size and mass of the object become larger, which can also be related to the stochastic stress-energy tensor for matter and radiation present in some regions of the universe. It has been shown by Sinha and Roy (1987) that the fluctuation of the space-time metric can give rise to a term like λ, i.e., the cosmological constant in the modified Friedman equation.

In this model the metric tensor of the space-time is assumed to be decomposed as

$$g^{\mu\nu} = \bar{g}^{\mu\nu} + \delta g^{\mu\nu} \tag{14}$$

where $\delta g^{\mu\nu}$ is considered as the fluctuating part, very small compared to $\bar{g}^{\mu\nu}$, the average part. In the same way the matter-energy tensor can be decomposed into two parts,

$$T^{\mu\nu} = \bar{T}^{\mu\nu} + \delta T^{\mu\nu} \tag{15}$$

where $\bar{T}^{\mu\nu}$ describes the global motion of matter characterized by a large scale L and period of time T, and the term $\delta T^{\mu\nu}$ is due to the turbulent motion of matter characterized by a small scale λ and short period τ; $T^{\mu\nu}$ is the mean value.

Now the Einstein equation is

$$R_{\mu\nu}=8\pi G(T_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}g^{\rho\sigma}T_{\rho\sigma})=8\pi GT^{*}_{\mu\nu} \tag{16}$$

where the stochasticity enters through the extra term of the energy-momentum tensor. In terms of the affine connection, the Ricci tensor on the left-hand side of equation (16) is

$$R_{\mu\nu}=\Gamma^{\alpha}_{\mu\alpha,\nu}-\Gamma^{\alpha}_{\mu\nu,\alpha}-\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha}-\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} \tag{17}$$

when the affine connection itself is defined in terms of the metric tensor by

$$\Gamma^{\rho}_{\mu\nu}=\tfrac{1}{2}g^{\rho\sigma}(g_{\sigma\mu,\nu}+g_{\sigma\nu,\mu}+g_{\mu\nu,\sigma}) \tag{18}$$

Denoting the average values with respect to the statistical ensemble by a bar, equation (18) turns into

$$\begin{aligned}\bar{\Gamma}^{\rho}_{\mu\nu}&=\langle\Gamma^{\rho}_{\mu\nu}\rangle\\ \langle R_{\mu\nu}\rangle&=\bar{\Gamma}^{\alpha}_{\mu\alpha,\nu}-\bar{\Gamma}^{\alpha}_{\mu\nu,\alpha}+\bar{\Gamma}^{\alpha}_{\mu\beta}\bar{\Gamma}^{\beta}_{\nu\alpha}\\ &\quad-\bar{\Gamma}^{\alpha}_{\nu\alpha}\bar{\Gamma}^{\beta}_{\alpha\beta}+\langle\gamma^{\alpha}_{\mu\beta}\gamma^{\beta}_{\nu\alpha}\rangle-\langle\gamma^{\alpha}_{\mu\nu}\gamma^{\beta}_{\alpha\beta}\rangle\end{aligned} \tag{19}$$

The tensor $\gamma^{\rho}_{\mu\nu}=\Gamma^{\rho}_{\mu\nu}-\bar{\Gamma}^{\rho}_{\mu\nu}$ relates to fluctuations in the gravitational force field with zero mean value $\langle\gamma^{\rho}_{\mu\nu}\rangle=0$ as a consequence of the definition $\bar{\Gamma}^{\rho}_{\mu\nu}=\langle\Gamma^{\rho}_{\mu\nu}\rangle$.

Again the mean value of the affine connection should be equal to the Christoffel symbols in the mean value of the metric tensor, i.e.,

$$\langle g_{\sigma\rho}\Gamma^{\rho}_{\mu\nu}\rangle=\tfrac{1}{2}(\bar{g}_{\sigma\mu,\nu}+\bar{g}_{\sigma\nu,\mu}-\bar{g}_{\mu\nu,\sigma}) \tag{20}$$

follows from the fact that the probability distribution over cosmologies will be Riemannian.

This is satisfied if

$$\langle(g_{\sigma\rho}-\bar{g}_{\sigma\rho})\Upsilon^{\rho}_{\mu\nu}\rangle=0 \tag{21}$$

Now the affine connection can be expressed in terms of Christoffel symbols in the mean value of the metric tensor as

$$\bar{\Gamma}^{\rho}_{\mu\nu}=\left\{{}_{\mu}{}^{\rho}{}_{\nu}\right\}=\tfrac{1}{2}\bar{g}^{\rho\alpha}(\bar{g}_{\sigma\mu,\nu}+\bar{g}_{\sigma\nu,\mu}-\bar{g}_{\mu\nu,\sigma}) \tag{22}$$

Then equations (17), (19), and (22) yield Einstein's equations for the mean value of the metric tensor,

$$\begin{aligned}&\left\{{}_{\mu}{}^{\alpha}{}_{\alpha}\right\}_{,\nu}-\left\{{}_{\mu}{}^{\alpha}{}_{\nu}\right\}_{,\alpha}+\left\{{}_{\mu}{}^{\alpha}{}_{\beta}\right\}\left\{{}_{\nu}{}^{\beta}{}_{\alpha}\right\}-\left\{{}_{\mu}{}^{\alpha}{}_{\nu}\right\}\left\{{}_{\alpha}{}^{\beta}{}_{\beta}\right\}\\ &\quad=8\pi G\bar{T}^{*}_{\mu\nu}\end{aligned} \tag{23}$$

where

$$\bar{T}^*_{\mu\nu}=\langle T^*_{\mu\nu}+(8\pi G)^{-1}(\gamma^{\alpha}_{\mu\beta}\gamma^{\beta}_{\nu\alpha}-\gamma^{\alpha}_{\mu\nu}\gamma^{\beta}_{\alpha\beta})\rangle \tag{24}$$

For simplicity, let us take the specialized stochastic space-time metric which are conformally related so that the metric tensor will be of the form

$$g_{\mu\nu}=\exp[\phi]\bar{g}_{\mu\nu} \tag{25}$$

where ϕ is a stochastic scalar field.

Then, as $\bar{g}_{\mu\nu}=\langle g_{\mu\nu}\rangle$, we get

$$\langle\exp[\phi]\rangle=1 \tag{26}$$

at all space-time points.

But the normalization condition implies that

$$\langle\exp[\phi]\phi_{,\mu}\rangle=0 \tag{27}$$

where $\phi_{,\mu}$ denotes the space-time derivative of ϕ.

Now, by putting (25) into (21) and using the definition (22), we get

$$\Gamma^{\rho}_{\mu\nu}=\{_{\mu}{}^{\rho}{}_{\nu}\}+\tfrac{1}{2}(\phi_{,\mu}{}^{\delta}{}_{\nu}{}^{\rho}+\phi_{,\nu}{}^{\delta}{}_{\mu}{}^{\rho}-\bar{g}_{\mu\nu}\bar{g}^{\rho\sigma}\phi_{,\sigma}) \tag{28}$$

Hence the probability distribution will be Riemannian with

$$\bar{\Gamma}^{\rho}_{\mu\nu}=\{_{\mu}{}^{\rho}{}_{\nu}\}$$

if and only if the homogeneity condition is satisfied:

$$\langle\phi_{,\mu}\rangle=0$$

Now it is clear that fluctuations in the gravitational field will be given by

$$\gamma^{\rho}_{\mu\nu}=\tfrac{1}{2}(\phi_{,\mu}{}^{\delta}{}_{\nu}{}^{\rho}+\phi_{,\nu}{}^{\delta}{}_{\mu}{}^{\rho}-\bar{g}_{\mu\nu}\bar{g}^{\rho\sigma}\phi_{,\sigma}) \tag{29}$$

Again the mean value of the stochastic stress-energy tensor is assumed to take the generic form

$$\langle T_{\mu\nu}\rangle\approx\langle\rho+P\rangle u_{\mu}u_{\nu}+\langle P\rangle\bar{g}_{\mu\nu} \tag{30}$$

where u_{μ} is a normalized timelike eigenvector of $\langle T_{\mu\nu}\rangle$. Let us now calculate $T^*_{\mu\nu}$:

$$\begin{aligned}\langle T^*_{\mu\nu}\rangle &=\langle\Gamma_{\mu\nu}\rangle-\tfrac{1}{2}\bar{g}_{\mu\nu}\bar{g}^{\rho\sigma}\langle\Gamma_{\rho\sigma}\rangle\\ &=\langle P+\rho\rangle u_{\mu}u_{\nu}+\langle P\rangle\bar{g}_{\mu\nu}-\tfrac{1}{2}\bar{g}_{\mu\nu}\bar{g}^{\rho\sigma}\\ &=(\langle P+\rho\rangle u_{\rho}u_{\sigma}+\langle P\rangle\bar{g}_{\rho\sigma})\\ &=(P+\rho)u_{\mu}u_{\nu}+\langle P\rangle\bar{g}_{\mu\nu}-\tfrac{1}{2}\bar{g}_{\mu}\langle P+\rho\rangle u^{\sigma}u_{\sigma}\\ &\quad-\tfrac{1}{2}\bar{g}_{\mu\nu}\bar{g}^{\rho\sigma}\bar{g}_{\rho\sigma}\langle P\rangle\end{aligned} \tag{31}$$

Putting $u^\sigma u_\sigma = -1$ and $\bar{g}^{\rho\sigma}\bar{g}_{\rho\sigma} = 4$, we find that equation (31) becomes

$$\langle T^*_{\mu\nu}\rangle \approx \langle P+\rho\rangle u_\mu u_\nu + \tfrac{1}{2}\langle \rho - P\rangle \bar{g}_{\mu\nu} \tag{32}$$

But since the correlation tensor $\langle \phi_{,\mu}\phi_{,\nu}\rangle$ is nonnegative definite and the only preferred direction at a space-time point is given by u_μ (time like), we must have

$$\langle \phi_\mu \phi_{,\nu}\rangle = A(u_\mu u_\nu + \xi \bar{g}_{\mu\nu}) \tag{33}$$

for a certain nonnegative scalar function A and ξ (≤ 1). So, from (32) and (33) it follows that

$$\begin{aligned}\langle \gamma^\alpha_{\mu\beta}\gamma^\beta_{\nu\sigma} - \gamma_\mu{}^\alpha{}_\nu \gamma_\alpha{}^\beta{}_\beta\rangle &= \tfrac{1}{2}[\bar{g}_{\mu\nu}\langle \phi_{,\alpha}\phi^\alpha{}_{,}\rangle - \langle \phi_{,\mu}\phi_{,\nu}\rangle] \\ &= \tfrac{1}{2}A[(3\xi - 1)\bar{g}_{\mu\nu} - u_\mu u_\nu]\end{aligned} \tag{34}$$

Using (33), the effective source tensor $\bar{T}^*$ is given by

$$\begin{aligned}\bar{T}^*_{\mu\nu} &= [\langle\rho\rangle + \langle P\rangle - (16\pi G)^{-1}]u_\mu u_\nu + \tfrac{1}{2}[\langle\rho\rangle - \langle P\rangle + (8\pi G)^{-1}(3\xi - 1)A]\bar{g}_{\mu\nu} \\ &= (\bar{\rho} - \bar{P})u_\mu u_\nu - \tfrac{1}{2}(\bar{\rho} - \bar{P})\bar{g}_{\mu\nu}\end{aligned} \tag{35}$$

where the effective density and pressure are given by

$$\bar{\rho} = \langle\rho\rangle + 3(16\pi G)^{-1}(\xi - \tfrac{1}{2})A \tag{36}$$

$$\bar{P} = \langle P\rangle + (16\pi G)^{-1}(\tfrac{1}{2} - 3\xi)A \tag{37}$$

This change in the equation of state will alter the solution to the Friedman equation

$$\frac{\ddot{R}}{R} = -4\pi G(\tfrac{1}{3}\bar{\rho} + \bar{P}) \tag{38}$$

$$\left(\frac{\dot{R}}{R}\right)^2 \frac{K}{R^2} = \tfrac{8}{3}\pi G\bar{\rho} \tag{39}$$

where $K = 0, \pm 1$.

Using the values of $\bar{\rho}$ and P from (36) and (37), we get the modified Friedman equation as

$$(\ddot{R}/R)^2 + K/R^2 = \tfrac{8}{3}\pi g\langle\rho\rangle + \tfrac{1}{2}(\xi - \tfrac{1}{2})A \tag{40}$$

But $4\pi\langle\rho\rangle R^3 = c$ (constant), so

$$(\dot{R})^2 + K - \tfrac{1}{2}(\xi - \tfrac{1}{2})AR^2 = 8\pi Gc/3R \tag{41}$$

or

$$(\dot{R})^2 + K - \Lambda R^2 = 8\pi Gc/3R \tag{42}$$

where

$$\Lambda=\tfrac{1}{2}(\xi-\tfrac{1}{2})A \tag{43}$$

This equation is identical to the Eddington–Lemaître model if we put $\Lambda=\frac{1}{2}(\xi-\frac{1}{2})A$. If $\xi>\frac{1}{2}$, $\bar{\rho}$ must be greater than $\langle\rho\rangle$, the nonnegative incremental change in $\dot{R}/R$ being given as $\frac{1}{2}\xi A$. Again, if $A=0$, then $\langle\phi_{,\mu}\phi_{,\nu}\rangle=0$, so that the correlation between $\phi_{,\mu}$ and $\phi_{,\nu}$ vanishes and $\Lambda=0$. It is clear that, if we take the stochasticity of the space-time metric as a prior notion, then obviously we obtain $\Lambda\neq 0$ for $\xi\neq 1$ and $A>0$. Now $\Lambda\neq 0$ denotes the empty space producing the same gravitational field as when the space contains matter with mass density $\phi_\Lambda=\varepsilon_\Lambda/c^2$, energy density $\varepsilon_\Lambda=c^2\Lambda/8\pi G$, and pressure $p_\Lambda=-\varepsilon_\Lambda$. From (36) we write $\bar{\rho}$ as $\bar{\rho}=\langle\rho\rangle+(3/16\pi G)(\xi-\frac{1}{2})A$. By calculating the average of $T_{\mu\nu}$ of the vacuum with the stochastic metric tensor we get

$$\begin{aligned}\langle T^{\text{vac}}_{\mu\nu}\rangle&=\frac{A}{2}[(3\xi-1)\bar{g}_{\mu\nu}-u_\mu u_\nu]\\&=\frac{A}{2}(3\xi-1)[\bar{g}_{\mu\nu}-u_\mu u_\nu(3\xi-1)]\\&=\rho_{\text{vac}}G_{\mu\nu}\end{aligned} \tag{44}$$

where

$$G_{\mu\nu}=\bar{g}_{\mu\nu}-u_\mu u_\nu\frac{1}{3\xi-1} \tag{45}$$

This metric tensor should be considered as the metric tensor of the vacuum or that of the background space-time. Since $\langle T^{\text{vac}}_{\mu\nu}\rangle$ is nonnegative definite, the metric tensor $G^{\text{vac}}_{\mu\nu}$ must be positive definite. We have already mentioned that in classical electrodynamics Schwinger *et al.* constructed the photon propagator containing the phenomenological information concerning the medium, where

$$D^{\mu\nu}_{+}(x-x')=\frac{\mu}{c}\left[g^{\mu\nu}+\left(1-\frac{1}{n^2}\right)\eta^\mu\eta^\nu\right]D(x-x') \tag{46}$$

$\eta^\mu=(0,1)$ is the unit timelike vector when the medium is at rest and n represents the refractive index of the medium. The last equation may be rewritten as

$$D_{+\mu\nu}(x-x')=\frac{\mu}{c}\,G_{\mu\nu}D(x-x')$$

where

$$G_{\mu\nu}=g_{\mu\nu}+\left(1-\frac{1}{n^2}\right)\eta^\mu\eta^\nu \tag{47}$$

which may be considered as the effective metric tensor of the vacuum with $n \neq 1$. Now it is obvious that there is a striking similarity between the metric for the cosmological vacuum and the effective metric tensor in classical electrodynamics.

Comparing (45) and (47), we get

$$1-\frac{1}{n^2}=\frac{1}{3\xi-1} \qquad \text{or} \qquad n^2=1-\frac{1}{3\xi} \tag{48}$$

The refractive index n may be real or imaginary, depending on the value of ξ as follows:

Case I. $\frac{1}{2}<\xi\leq 1$. We see from the relation (48) that $n^2<1$ and positive. This indicates that when the universe is not closed ($k=+1$) the refractive index of the vacuum n becomes less than one:

$$m_\gamma^2=\hbar^2\omega^2(1-n^2)-\frac{\sigma^2\hbar^2}{4n^2(\varepsilon_0\kappa_0)^2}$$

This implies that for these values of ξ the photon will be massive. But there will be no significant shift in the frequency (i.e., red shift). This is due to the fact that for real n (and $n<1$) the damping factor $e^{-\omega\beta z}$ with $\beta=\frac{1}{2}(\sigma/n)^2(1/\omega^2)$ will be very large and the medium behaves almost like a nonconducting medium.

Case II. $\xi=\frac{1}{2}$. This means $\Lambda=0$, i.e., the cosmological constant vanishes. In this case, $n^2=\frac{1}{3}$.

Case III. $\frac{1}{3}<\xi<\frac{1}{2}$. In this range Λ will be negative but the refractive index n will still be real and positive.

Case IV. $0<\xi<\frac{1}{3}$. The refractive index n will be imaginary and the vacuum behaves like a conducting dielectric medium. So a red shift will occur when a photon passes through this vacuum. In such a case the photon mass m_γ will be large and for very small ω [equation (6)]

$$m_\gamma\simeq\frac{\sigma}{2n\varepsilon_0\kappa_e} \tag{49}$$

Comparing the relation for m_γ with equation (8), we have

$$H=\frac{\sigma}{2\pi n\varepsilon_0\kappa_e} \tag{50}$$

For example, let us take $\xi=\frac{1}{4}$ (i.e., within the range $0<\xi<\frac{1}{3}$). Then $\sigma\simeq 2.2\times 10^{-17}\ \text{sec}^{-1}$ taking $H=1.5\times 10^{-17}\ \text{sec}^{-1}$. Hence the conductivity of the vacuum might be experimentally verified as being of the order of 2.2×10^{-17} sec^{-1}

4. POSSIBLE IMPLICATIONS

We have constructed a model of a vacuum which is dielectric in nature with refractive index $n^2 = 1 - 1/(3\xi)$ with $0 < \xi \leq 1$. This vacuum can behave like a conducting medium or a nonconducting one, depending on the range of values of ξ. The cosmological red shift can be explained by taking this vacuum as some sort of conducting medium. But on the microlevel it is generally believed that due to the nondissipative nature of quantum fluctuations the vacuum should behave at least not like a conducting medium. Let us concentrate first on the case of the behavior of our model of a vacuum on the cosmological scale.

4.1. Cosmological Vacuum

The cosmological vacuum is considered as a dielectric medium with complex refractive index. Here the nonzero photon mass is closely related to the dissipative mechanism of the vacuum. This mechanism of dissipation is understandable within the general theory of relativity in the following manner.

If we consider the propagation of very light particles, say photons, in a stochastic medium characterized by a fluctuating metric tensor $g_{\mu\nu}$, then general relativity implies that action and reaction between the $g_{\mu\nu}$ field and any moving object (say photon) is characterized by an energy-momentum distribution $T_{\mu\nu}$ in the Einstein equation. Here, the gravitational red shift is being induced by the fluctuation of the metric tensor on the passing photon motion. In a sense it is the backreaction of the $g_{\mu\nu}$ field on the photon. Usually the backreaction is very small and has been generally neglected in the literature.

It is to be mentioned that Vigier (1990) considered the cosmological vacuum as some sort of dielectric medium with complex refractive index and also described some kind of vacuum dissipative mechanism associated with a nonzero rest mass of the photon. The point he mentioned is that the quantum fluctuations are, in fact, real statistical fluctuations which reflect the basic stochastic character of Dirac's aether and hence deserve a critical analysis. On the contrary, Smolin (1986) considered that the quantum fluctuations are unlike other statistical fluctuations and they are supposed to be nondissipative in character, in contrast to other statistical fluctuations. So if we want to build up a stochastic theory of microparticles [Nelson's (1985) mechanics] or the stochastic interpretation of quantum mechanics developed by Vigier (1990) with a fluctuating vacuum, it is necessary to reanalyze the structure of the vacuum in the following way.

4.2. Quantum Fluctuation and Model of the Vacuum

From (50) it appears that for $n^2 \ll 1$ ($\frac{1}{3} < \xi \leq 1$), the conductivity coefficient σ will be very small. Then the vacuum considered as a dielectric medium should be treated as a poorly conducting medium. In the limiting case it behaves like an insulator and the nonzero mass of the photon $m_\gamma \to 0$. In fact, this is consistent with the fact that in quantum electrodynamics we take the vanishing rest mass of the photon and so gauge invariance is retained.

Now if we consider the propagation of the photon in this type of stochastic insulating medium, the dissipation will be negligibly small. This is quite unlike other statistical fluctuations in physics. It is interesting to note that Dohrn and Guerra (1985) established a connection between the kinetic metric and the Brownian metric as follows:

$$\eta^{\mu\nu} = 2u^\mu u^\nu - g^{\mu\nu} \tag{51}$$

where $g^{\mu\nu}$ is not necessarily a positive-definite metric tensor in the Lagrangian

$$L = \tfrac{1}{2} m g_{\mu\nu} \dot{q}^\mu(t) \dot{q}^\nu(t) - V[q, (t)] \tag{52}$$

in a differentiable manifold endowed with the metric tensor $g_{\mu\nu}$ and $\eta^{\mu\nu}$ is a Brownian metric which is positive definite.

From (35) we can write the effective stress tensor

$$\bar{T}^*_{\mu\nu} = (\bar{\rho} + \bar{P}) u_\mu u_\nu + \tfrac{1}{2}(\bar{\rho} - \bar{P}) \bar{g}_{\mu\nu}$$

where

$$\bar{\rho} = \langle \rho \rangle + \frac{3}{16\pi G}(\xi - \tfrac{1}{2}) A$$

$$\bar{P} = \langle P \rangle + \frac{1}{16\pi G}(\tfrac{1}{2} - 3\xi) A$$

$$\bar{\rho} + \bar{P} = \langle \rho \rangle + \langle P \rangle - \frac{1}{16\pi G} A$$

$$\bar{\rho} - \bar{P} = \langle \rho \rangle - \langle P \rangle + \frac{A}{8\pi G}(3\xi - 1)$$

So

$$\bar{T}^{*}_{\mu\nu}=\left(\langle\rho\rangle+\langle P\rangle-\frac{1}{16\pi G}A\right)u_\mu u_\nu+\tfrac{1}{2}\left(\langle\rho\rangle-\langle P\rangle+\frac{A}{8\pi G}(3\xi-1)\right)\bar{g}_{\mu\nu}$$

$$=\langle T^{\text{matter}}_{\mu\nu}\rangle-\frac{A(3\xi-1)}{16\pi G}\left(\frac{1}{3\xi-1}u_\mu u_\nu-\bar{g}_{\mu\nu}\right)$$

$$=\langle T^{\text{matter}}_{\mu\nu}\rangle+\langle T^{\text{vacuum}}_{\mu\nu}\rangle$$

where

$$\langle T^{\text{vacuum}}_{\mu\nu}\rangle=\frac{A(3\xi-1)}{16\pi G}\left(\frac{1}{3\xi-1}u_\mu u_\nu-\bar{g}_{\mu\nu}\right)$$

$$=(-\rho_{\text{vac}})(-G^{\text{vac}}_{\mu\nu})=e^{*}_{\text{vac}}\cdot\overset{*}{G}{}^{\text{vac}}_{\mu\nu}$$

where

$$\overset{*}{G}{}^{\text{vac}}_{\mu\nu}=\frac{1}{3\xi-1}u_\mu u_\nu-\bar{g}_{\mu\nu}$$

$\bar{g}_{\mu\nu}$ is the usual Riemannian metric tensor and $0<\xi\leq 1$. For $\xi=\frac{1}{2}$,

$$\overset{*}{G}{}^{\text{vac}}_{\mu\nu}=2u_\mu u_\nu-\bar{g}_{\mu\nu} \tag{53}$$

As $\langle T^{\text{vac}}_{\mu\nu}\rangle$ is the average of the stress tensor for the vacuum, and $\langle\partial\phi\,\partial\phi/\partial x_\mu\,\partial x_\nu\rangle$ is nonnegative definite, $\overset{*}{G}{}^{\text{vac}}_{\mu\nu}$ is positive definite even if $g_{\mu\nu}$ is not positive definite. So for $\xi=\frac{1}{2}$, we get exactly the same metric tensor for the vacuum that Dohrn and Guerra (1985) got for the conservative diffusion process. It is very important to notice that for $\xi=\frac{1}{2}$, $\Lambda=0$, i.e., the cosmological constant vanishes, so the fluctuation of the space-time metric which does not give rise to any extra term (like the cosmological constant term) in the Einstein equation may be responsible for conservative diffusion considered in Nelson's framework.

Thus the above analysis indicates that not only can microscopic phenomena be explained by considering the fluctuation of the space-time metric, but also some macroscopic phenomena such as the cosmological constant problem can be explained consistently. The most startling point of our approach is that even some cosmological models can be tested in laboratory experiments by measuring the conductivity coefficient of the vacuum. It should be noted that we can get an effect nonzero mass of the photon only for a certain range of frequencies. Moreover, the propagation velocity of the fluctuation or the disturbance of the medium, i.e., v_p, is less than the speed of light. So no superluminal transmission is allowed in this vacuum. These considerations might have great significance in quantum mechanics, which will be examined in subsequent publications.

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REFERENCES

De Broglie, L., and Vigier, J. P. (1972). *Physical Review Letters*, **28**, 1001.
Dohrn, D., and Guerra, F. (1985). *Physical Review D*, **31**, 2531.
Fuli, L. (1975). *Nuovo Cimento*, **3**, 289.
Marochnik, L. S. (1968). *Soviet Astronomy*, **AJ12**, 171.
Nelson, E. (1985). *Quantum Fluctuations*, Princeton University Press, Princeton, New Jersey.
Schwinger, J., Tsai, W., and Erber, T. (1976). *Annals of Physics*, **96**, 303.
Sinha, M., and Roy, S. (1987). *Nuovo Cimento*, **100B**(6), 709.
Smolin, L. (1986). *Classical and Quantum Gravity*, **3**, 347, and references therein.
Vigier, J. P. (1990). *IEEE Transactions on Plasma Science*, **18**(Feb).